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Charles Roberts, 6 Bramerton Street, King's Road, Chelsea, S.W. (proposed by Adam Hilger).

Mr. J. E. Drower, Mr. W. B. Gibbs, and Mr. H. P. Hollis were appointed Auditors of the Treasurer's accounts for the year 1894.

One hundred and five presents were announced as having been received since the last meeting, including, amongst others—

A series of sixteen memoirs from the *Comptes Rendus*, &c., by M. Deslandres, presented by the Author, and copies on glass of nine spectrum photographs of the solar prominences, &c.; Détermination des constantes pour la réduction des clichés pris à Helsingfors, pour la construction du carte du Ciel, par A. Donner, presented by the Author; Radiant Suns, by Agnes Giberne, presented by the Author; Three volumes, containing a series of drawings of Sun-spots, presented by the Rev. F. Howlett; Publications of von Kuffner's Observatory, Vienna, vol. iv., presented by the Observatory; Lund Observatory Zone Observations + 35° to + 40°, presented by the Observatory; Scheiner's Treatise on Astronomical Spectroscopy, translated, revised, and enlarged by E. B. Frost, presented by Professor Frost; Scheiner, Untersuchungen über den Lichtwechsel Algols, presented by Dr. L. Becker.

A Re-determination of the Epoch Correction and Mean Motion for Hansen's Lunar Tables from Meridian Observations of the Moon, 1750–1892. E. J. Stone, M.A., F.R.S., Radcliffe Observer.

In the present paper it is proposed to apply the views which I hold of the physical bases of the measures of time adopted in practical astronomical work to a re-determination of the epoch constant and mean motion in Hansen's Lunar Tables.

The results of all the meridian observations of the Moon made at Greenwich between 1750 and 1882, and those made at the Radcliffe Observatory, Oxford, from 1880 to 1892, will be included in the discussion.

The residuals give in column (3) of the following table have been taken from Airy's determinations of the mean errors in longitude of Damoiseau's Tables, *Memoirs*, vol. xxix., but they have been corrected for those parts of the differences of the Greenwich sidereal times which correspond to the beginning of the "unit day," which have usually been acknowledged and allowed for, in passing from the data adopted in Carlini's Tables to the data adopted in Le Verrier's Tables; and these corrections have been carried into all the residuals given in column (3).

The residuals, 1847–1892, have been obtained from direct

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comparisons between Hansen's Lunar Tables and observations made in the usual manner.

My views on time-measurement are the direct mathematical consequences of the necessary continuity of our measures of time and of length, and the assumed truth of the three following working hypotheses:—

1. The law of universal gravitation.
2. The constancy of the Sun's mass.
3. The uniformity of the rotation of the Earth around its axis.

Sensible constancy during the whole period over which our investigations extend may be understood instead of absolute constancy.

These working hypotheses would have to be abandoned if it could be shown that, subject to the necessary continuity of our measures of time and of length, they led to results inconsistent with the facts of observation.

But, subject to the truth of these working hypotheses, the symbol, f, which expresses the accelerating effects of the Sun's mass, adopted as the unit of mass, at the unit of length, in the unit of time, cannot be changed in evaluating the gravitational effects of the different masses so long as the same units of length and of time are adopted.

To assume otherwise, and that different numerical values, f_1, f_2, f_3, &c., might be assigned to f in the investigation of the disturbing effects of the planetary masses on the motions of *Mercury*, *Venus*, the Earth, and the Moon, would be to assume, in practice, that the law of universal gravitation was not true. And the more accurately the differential equations of motion were formed and integrated with the use of different values f_1, f_2, &c., for f and the more exact the observations, the more evident would become the divergency between the mathematical results thus erroneously obtained and the facts of observation. Different numerical values can be assigned to f in our mathematical work by the adoption of different units of length and of time, but not otherwise. The numerical values which can be assigned to f are necessarily connected with the different units which we can adopt in such a way that $f \propto$ directly as the square of the interval of time which is adopted as the unit, and inversely as the cube of the length which is adopted as the unit.

But if ω denotes the constant angular velocity of the Earth about its axis, the numerical values which can be assigned to ω in our mathematical work are subject to the condition that they vary directly as the interval of time which is adopted as the unit. It follows, therefore, that, whatever unit of time we may adopt, the numerical values which are assignable to the $\sqrt{f}$ and ω in our mathematical work must be such that the ratio $\sqrt{f} : \omega$ is an absolute constant so long as the same unit of length is adopted. If, therefore, in the investigations of the heliocentric motion of the centre of gravity about the Sun we eliminate the

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quantity f from our equations by the condition that $f = a^3 n^2$, where a is a linear quantity which may, by a proper selection of the unit of length, be made $= 1$, and n is the coefficient of t in the expression of the heliocentric longitude of the Earth, it follows that the ratio $\omega : n$ is an absolute constant for any possible units of length and of time that we can adopt; and that the angles n and ω can only be found from observation subject to the condition $\frac{\omega}{n}$ is an absolute constant, and we can no more change n in our equations without an alteration of the unit of time than we can change ω or f.

The necessary conditions to preserve the same value of f in the lunar and solar theories when subsidiary conditions such as $f = a^3 \mu^2$ are introduced, and μ is taken as the coefficient of t in the expression of the geocentric longitude of the Moon, are usually satisfied by the introduction of the condition that the ratio $\mu : n :: 1 : m$, where m is an absolute constant; and the results thus obtained should be exact, provided the numerical value ultimately assigned to μ is found on the scale of time-measurement fixed by the numerical value ultimately assigned to n; and the value of m is thus deduced from the ratio of two angular velocities referred to the same unit.

And what is true of ω and n will be equally true of Ω and N, if Ω denotes the mean motion of the meridian in right ascension measured from the true equinox, and N the coefficient of t in the expression for the geocentric longitude of the Sun, also measured from the true equinox. We shall have, therefore, in order to find accurate values of Ω and N from observation to introduce the condition that the ratio $\Omega : \mathrm{N}$ is an absolute constant, whatever unit of length and of time we may adopt.

The differences between my results and those which have hitherto met with acceptance entirely turn upon the point whether the condition $\frac{\omega}{n}$, an absolute constant, is a fundamental condition which must be satisfied in order that our gravitational investigations can be exact after we have introduced into them the condition $f = a^3 n^2$ or $f = n^2$, and $\mu : n :: 1 : m$, m being a constant.

The condition $\frac{\Omega}{\mathrm{N}}$, an absolute constant, leads to the condition that Ω may be expressed under the form

$$\left(2\pi \frac{\mathrm{N}}{\mathrm{N}_0} + \mathrm{N}\right) = 2\pi + \mathrm{N} + 2\pi \frac{\mathrm{N} - \mathrm{N}_0}{\mathrm{N}_0},$$

which cannot, therefore, $= 2\pi + \mathrm{N}$ unless N happens to be identically equal to N_0, a definite angle for a definite epoch. For any other adopted angle N, we have, therefore,

$$\Omega_1 = 2\pi \frac{\mathrm{N}_1}{\mathrm{N}_0} + \mathrm{N}_1 = 2\pi + \mathrm{N}_1 + 2\pi \frac{\mathrm{N}_1 - \mathrm{N}_0}{\mathrm{N}_0},$$

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and therefore

$$\Omega_1 - \Omega = N_1 - N + 2\pi \frac{N_1 - N}{N_0}.$$

Astronomers have hitherto duly allowed for the effects of the term $N_1 - N$, but have overlooked the term

$$2\pi \frac{N_1 - N}{N_0}$$

in the difference $\Omega_1 - \Omega$; and *it was in this way that Hansen determined, from a discussion of observations,* the epoch correction and mean motion adopted in his tables which, if my views are correct, are necessarily defective, because results obtained in terms of one scale of time-measurement fixed by an adopted angle, N, were compared with observational results to obtain which another scale of time-measurement fixed by an adopted angle, N_1, had been employed.

The corrections, δS, given in column (7) of the following table are the effects on the tabular mean longitudes due to the neglect of the terms

$$2\pi \frac{N_1 - N}{N_0},$$

computed from the values N_1 and N used by astronomers before and after 1864. The effects of these terms are certainly important; the only question at issue is whether they are, or are not, necessary if the continuity of our measure of time is to be preserved. I maintain, of course, that they are necessary, and I have simply traced the consequences of their application.

The difference between Hansen's and Le Verrier's values of N is but slight, and can easily and directly be allowed for by the transformation of Hansen's expressions, so far as they depend on the Sun's position, before proceeding to the determination of the constants involved in the periodical terms with the corrected values of the epoch constant ϵ and the mean motion μ.

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Reference No.	(1) Years grouped.	(2) Mean Year.	(3) Tabular *minus* Observed D to 1851 H 1848–1892.	(4) Correction to reduce D to H in Mean Longitude.	(5) Correction to D for Hansen's Long Inequalities.	(6) Sum of the three preceding Columns and Hansen − Observed when the Corrections δS are neglected.	(7) Correction for δS usually neglected.	(8) Sum of the two preceding Columns, and Hansen − Observed afforded by Errors in Epoch and Mean Motion.	(9) Hansen *minus* Observed with New Epoch and Mean Motion.
			"	"	"	"	"	"	"
1	1750–1759	1754·5	+ 3·13	−27·83	+26·62	+ 1·92	−87·90	−85·98	−0·93
2	1755–1764	1759·5	+ 1·31	−25·03	+25·87	+ 2·15	−83·89	−81·74	−0·54
3	1760–1768	1764·0	+ 0·23	−22·51	+24·84	+ 2·56	−80·28	−77·72	+0·01
4	1765–1773	1769·0	− 1·52	−19·69	+23·29	+ 2·08	−76·27	−74·19	−0·31
5	1769–1778	1773·5	− 3·14	−17·14	+21·56	+ 1·28	−72·65	−71·37	−0·96
6	1774–1782	1778·0	− 3·51	−14·57	+19·52	+ 1·44	− 9·04	−67·60	−0·65
7	1779–1787	1783·0	− 3·16	−11·72	+16·94	+ 2·06	−65·03	−62·97	+0·12
8	1783–1791	1787·0	− 3·89	− 9·44	+14·65	+ 1·32	−61·81	−60·49	−0·48
9	1788–1796	1792·0	− 4·62	− 6·56	+11·55	+ 0·37	−57·80	−57·43	−1·27
10	1792–1801	1796·5	− 2·91	− 3·96	+ 8·57	+ 1·70	−54·19	−52·49	+0·20
11	1797–1805	1801·0	− 1·03	− 1·36	+ 5·45	+ 3·06	−50·58	−47·52	+1·70
12	1802–1810	1806·0	− 1·30	+ 1·55	+ 1·85	+ 2·10	−46·56	−44·46	+0·91
13	1806–1815	1810·5	− 1·11	+ 4·18	− 1·46	+ 1·61	−42·95	−41·34	+0·56
14	1811–1819	1815·0	+ 0·42	+ 6·81	− 4·77	+ 2·46	−39·34	−36·88	+1·56
15	1816–1824	1820·0	+ 1·19	+ 9·76	− 8·43	+ 2·52	−35·32	−32·80	+1·78

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Reference No.	(1) Years grouped.	(2) Mean Year.	(3) Tabular *minus* Observed D to 1851 H 1848–1892.	(4) Correction to reduce D to H in Mean Longitude.	(5) Correction to D for Hansen's Long Inequalities.	(6) Sum of the three preceding Columns and Hansen – Observed when the Corrections δS are neglected.	(7) Correction for δS usually neglected.	(8) Sum of the two preceding Columns, and Hansen – Observed affected by Errors in Epoch and Mean Motion.	(9) Hansen *minus* Observed with New Epoch and Mean Motion.
16	1820–1829	1824·5	″ + 0·90	″ + 12·41	″ – 11·64	″ + 1·67	″ – 31·71	″ – 30·04	″ + 1·08
17	1825–1833	1829·0	+ 0·93	+ 15·08	– 14·74	+ 1·27	– 28·10	– 26·83	+ 0·82
18	1830–1838	1834·0	+ 1·35	+ 18·06	– 17·99	+ 1·42	– 24·08	– 22·66	+ 1·14
19	1834–1842	1838·0	+ 0·77	+ 20·45	– 20·41	+ 0·81	– 20·87	– 20·06	+ 0·65
20	1839–1847	1843·0	– 0·40	+ 23·44	– 23·17	– 0·13	– 16·86	– 16·99	– 0·13
21	1843–1851	1847·0	– 2·13	+ 25·85	– 25·14	– 1·42	– 13·65	– 15·07	– 1·29
22	1848–1856	1852·0	– 1·50	...	...	– 1·50	– 9·63	– 11·13	– 1·20
23	1852–1860	1856·0	– 2·24	...	...	– 2·24	– 6·42	– 8·66	– 1·82
24	1857–1865	1861·0	– 1·89	...	...	– 1·89	– 2·50	– 4·39	– 1·40
25	1861–1869	1865·0	+ 0·85	...	...	+ 0·85	– 0·54	+ 0·31	+ 0·22
26	1866–1874	1870·0	+ 5·43	...	...	+ 5·43	0·00	+ 5·43	+ 1·49
27	1870–1878	1874·0	+ 7·90	...	...	+ 7·90	0·00	+ 7·90	+ 0·88
28	1875–1883	1879·0	+ 10·37	...	...	+ 10·37	0·00	+ 10·37	– 0·51
29	1879–1887	1883·0	+ 13·16	...	...	+ 13·16	0·00	+ 13·16	– 0·80
30	1884–1892	1888·0	+ 16·99	...	...	+ 16·99	0·00	+ 16·99	– 0·82

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A discussion of the residual errors in column (8), with an allowance for the fact that the mean annual residuals refer to the middle and not the commencement of the year, gives for the necessary corrections to epoch, ϵ, and mean motion, μ, the following values :—

$$\delta\epsilon = +49''\cdot994 + 0''\cdot385 = 50''\cdot379$$

$$\delta\mu = -0''\cdot7705$$

the epoch being 1800 Jan 0·0. G.M.T.

The residual errors, for the thirty equations from which the corrections to the epoch and mean motion have been deduced, are given in column (9).

But on account of the use of residuals indirectly deduced from comparisons made through Damoiseau's Tables, I have given as a check the mean annual residuals for all the years 1847–1892 for which I have results available from direct comparisons between Hansen's Tables and observations. It will be seen that, although the observations of the last forty years cannot have been used by Hansen in the deduction of data for the formation of his tables, the agreement between his results and those of observation are remarkably close. The mean of the residual errors in longitude for the years 1847–1892 is only —0″·32, and the mean error only 1″·17.

Mean Excess over Observation of the Moon's Tabular Place in Longitude for the years 1847 *to* 1892, *as computed from Hansen's Tables uncorrected and with the corrected Epoch and Mean Motion determined from the Meridian Observations of the Moon*, 1750 *to* 1892.

Hansen *minus* Observed
(for all the Annual Results available from direct comparison with Hansen's Tables).

Year.	Uncorrected.	Corrected.	Year.	Uncorrected.	Corrected.
	″	″		″	″
1847	+0·51	+1·04	1870	+ 4·65	+0·70
1848	−0·53	+0·04	1871	+ 6·71	+1·99
1849	−1·08	−0·48	1872	+ 7·05	+1·56
1850	−0·97	−0·34	1873	+ 7·95	+1·69
1851	−1·93	−1·27	1874	+ 8·96	+1·93
1852	−1·57	−0·87	1875	+ 9·51	+1·71
1853	−2·18	−1·46	1876	+ 9·45	+0·88
1854	−2·34	−1·59	1877	+ 8·91	−0·43
1855	−1·40	−0·62	1878	+ 7·93	−2·18
1856	−1·51	−0·69	1879	+ 9·29	−1·59
1857	−2·41	−1·56	1880	+10·71	−0·94
1858	−2·61	−1·73	1881	+10·32	−2·10
1859	−2·49	−1·57	1882	+12·45	−0·74
1860	−3·62	−2·67	1883	+14·71	+0·75
1861	−2·95	−1·97	1884	+14·65	−0·08
1862	−2·72	−1·71	1885	+15·20	−0·30
1863	−1·54	−0·49	1886	+15·34	−0·93
1864	+0·12	+0·80	1887	+15·70	−1·34
1865	+1·22	+1·13	1888	+17·68	−0·13
1866	+2·07	+1·21	1889	+17·37	−1·21
1867	+3·35	+1·72	1890	+18·02	−1·33
1868	+3·97	+1·57	1891	+19·30	−0·83
1869	+4·12	+0·95	1892	+19·69	−1·20

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The results here obtained are the converse of those which I have annually brought before the Society for some years. In those comparisons between Hansen's Tables and the observations of the Moon I have accepted Hansen's Tables with the epoch and mean motion which he found for observations made during a period when the data of Carlini's Solar Tables were employed; and I corrected the times, t, found as at present to reduce them to the scale for which the values t and μ had been found. In the present investigation I have conversely corrected the times determined, 1750–1864, and reduced them to the present scale of time-measurement, and then re-determined the epoch and mean motion on the scale at present adopted. But as this gives a period of at least forty additional years for the determination of the mean motion, the result may be expected to be more accurately found on its scale than that found by Hansen.

I have not, at present, attempted to correct the determination of the constants involved in the periodic terms. Such a step appears useless until the great fundamental principles at issue in these questions have been definitely settled. But if anyone will assume that the time as now found is absolutely correct, or that the tabular places are correctly referred to the meridian with Le Verrier's value of N, or, in other words, if

$$2\pi \, . \, \frac{N - N_0}{N_0} \, . \, t$$

is a negligible angle, and the corrections

$$\delta\epsilon = +50''{\cdot}379$$
$$\delta\mu = -0''{\cdot}7705$$

are carried into the mean longitude of Hansen's Tables, the tabular results thus obtained will agree very closely indeed with the results of observation.

I am engaged in examining the effects of the rejection of one of Hansen's long inequalities, but at present I can only say that if this were done the necessity for the corrections δS is equally apparent. The residual errors are not so small as before, but they are, perhaps, not much larger than are admissible if the old observations are affected, as may be the case, with sensible systematic errors.